

Non-relativistic M2-brane gauge theory and new superconformal algebra

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

JHEP04(2009)096

(<http://iopscience.iop.org/1126-6708/2009/04/096>)

[The Table of Contents](#) and [more related content](#) is available

Download details:

IP Address: 80.92.225.132

The article was downloaded on 03/04/2010 at 10:31

Please note that [terms and conditions apply](#).

Non-relativistic M2-brane gauge theory and new superconformal algebra

Yu Nakayama,^a Makoto Sakaguchi^b and Kentaroh Yoshida^c

^a*Berkeley Center for Theoretical Physics and Department of Physics,
University of California, Berkeley, California 94720-7300, U.S.A.*

^b*Okayama Institute for Quantum Physics,
1-9-1 Kyoyama, Okayama 700-0015, Japan*

^c*Kavli Institute for Theoretical Physics,
University of California, Santa Barbara, CA 93106, U.S.A.*

E-mail: nakayama@berkeley.edu, makoto_sakaguchi@pref.okayama.jp,
kyoshida@kitp.ucsb.edu

ABSTRACT: We study non-relativistic limits of the $\mathcal{N} = 6$ Chern-Simons-Matter theory that arises as a low-energy limit of the M2-brane gauge theory with background flux. The model admits several different non-relativistic limits and we find that the maximal supersymmetry we construct has 14 components of supercharges, which is a novel example of non-relativistic superconformal algebra in $(1 + 2)$ dimension. We also investigate the other limits that realize less supersymmetries.

KEYWORDS: Field Theories in Lower Dimensions, Extended Supersymmetry, Chern-Simons Theories, M-Theory

ARXIV EPRINT: [0902.2204](https://arxiv.org/abs/0902.2204)

Contents

| | | |
|----------|---------------------------------------|-----------|
| 1 | Introduction | 1 |
| 2 | Mass deformed ABJM model | 3 |
| 3 | Non-relativistic limit | 4 |
| 3.1 | Action | 4 |
| 3.2 | Bosonic symmetry | 7 |
| 3.3 | Supersymmetry | 9 |
| 3.4 | Summary of the superconformal algebra | 12 |
| 4 | Less SUSY limit | 12 |
| 4.1 | 8 SUSY limit | 12 |
| 4.2 | 4 SUSY limit | 15 |
| 4.3 | 0 SUSY limit | 16 |
| 5 | Discussion and summary | 16 |
| A | Spinor convention | 17 |
| B | Consistency of the truncation | 17 |

1 Introduction

The ubiquity of the Chern-Simons-Matter system has been much appreciated in recent studies of theoretical physics. On one corner of the theoretical physics, i.e. in string theory, the M2-brane mini-revolution [1–4] has created a novel class of gauge-gravity correspondences based on the Chern-Simons-Matter theory, and we believe that it will eventually bring us deeper understanding of the M-theory itself. On the other corner of the theoretical physics, i.e. in condensed matter physics, the Chern-Simons-Matter theory has been long known to give an indispensable tool to analyze the effective theory that appears in the quantum Hall effects.

The natural question that connects these two distinguishing branches of theoretical physics would be: Can we understand the quantum Hall effect from M2-brane gauge theory?¹ The question is much like whether we can understand the QCD from the string theory. Although it is true that the quantum Hall effect in the effective Chern-Simons-Matter system is not supersymmetric (like real QCD) and the rank of the gauge group is

¹Fractional quantum Hall effect has been discussed in [5] by using the edge states in the ABJM model [4] and other D-brane setups.

just Abelian, we expect that qualitative features of such a theory can be extracted from the non-relativistic limit of these M2-brane gauge theories.

For example, one can use the “Seiberg duality” of $\mathcal{N} = 2$ Chern-Simons-Matter theory [6, 7] to translate a level k $U(1)$ Chern-Simons theory with one fundamental matter multiplet to a level k $U(k)$ Chern-Simons theory with one fundamental matter multiplet coupled with a singlet supermultiplet. It may be possible to study the large k behavior from the string theory because the latter dual theory is strongly coupled from the gauge theory viewpoint.

However, the real hurdle in this scenario lies in the non-relativistic limit, which is the main scope of this paper. Even the supersymmetry (SUSY) can be completely broken in the limiting procedure, depending on the specific non-relativistic limit that we choose. It is furthermore not *a priori* obvious how many supersymmetries can be realized in a given non-relativistic conformal limit. We note that the complete classification of the non-relativistic superconformal algebra is still unavailable. Unlike the relativistic superconformal system, there seem a lot more possibilities. Indeed, we will find many in this paper.

In [8, 9], they studied the non-relativistic superconformal algebra embedded in one-dimensional higher dimensional relativistic superconformal algebra. This is a standard way to realize the bosonic counterpart: Schrödinger algebras inside a relativistic conformal algebra (or AdS algebra).² Some non-relativistic superconformal algebras we obtain in this paper are not included in their list. Furthermore, the explicit construction of the non-relativistic superconformal field theories is non-trivial even when the algebra is known (see [21–24] for some previous attempts from the field theory).

With this motivation, we study the non-relativistic limit of $\mathcal{N} = 6$ Chern-Simons-Matter theory [4] (known as ABJM model). The model is a candidate dual gauge theory for M2-branes in orbifold space. We introduce the background 4-form flux that yields the maximal supersymmetric mass deformation [25–27]. The non-relativistic limit of the theory gives a novel supersymmetric Chern-Simons-Matter theory with maximum 14 supercharges. We also obtain less supersymmetric limits, which include the supersymmetric theory without any superconformal charges (but invariant under the full bosonic Schrödinger group).

The organization of the paper is as follows. In section 2, we study the maximal supersymmetric mass deformation of the ABJM model. In section 3, we take the maximal supersymmetric non-relativistic limit of the mass-deformed ABJM model, and investigate the non-relativistic superconformal algebra. In section 4, we examine other possible non-relativistic limits, which yield less supersymmetric theories. In section 5, we give some discussions and conclude the paper. In appendix A, we have summarized our spinor convention in $(1 + 2)$ dimension. We discuss the consistency of the truncation in appendix B.

²Non-relativistic conformal algebra [10–13] is sometimes called Schrödinger algebra because it was originally found as the maximal symmetry of a free Schrödinger equation. See also [14–20] for further investigations.

2 Mass deformed ABJM model

ABJM model describes a low energy effective theory on the M2-branes probing the $\mathbf{C}_4/\mathbf{Z}_k$ orbifold. It is a $U(N) \times U(N)$ Chern-Simons quiver gauge theory with bi-fundamental matter fields. The model has the manifest $\mathcal{N} = 6$ superconformal symmetry (with 24 supercharges).

Our starting point is the relativistic action for the ABJM model given by

$$S = \int d^3x \left[\frac{k}{4\pi} \epsilon^{\mu\nu\lambda} \text{Tr} \left(A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda - \hat{A}_\mu \partial_\nu \hat{A}_\lambda - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda \right) - \text{Tr} D_\mu X_A^\dagger D^\mu X^A - i \text{Tr} \bar{\Psi}^A \gamma^\mu D_\mu \Psi_A - V_{\text{bos}} - V_{\text{fer}} \right], \quad (2.1)$$

where the bosonic potential is given by

$$V_{\text{bos}} = -\frac{4\pi^2}{3k^2} \text{Tr} \left(X^A X_A^\dagger X^B X_B^\dagger X^C X_C^\dagger + X_A^\dagger X^A X_B^\dagger X^B X_C^\dagger X^C + 4X^A X_B^\dagger X^C X_A^\dagger X^B X_C^\dagger - 6X^A X_B^\dagger X^B X_A^\dagger X^C X_C^\dagger \right), \quad (2.2)$$

while the fermionic potential is given by

$$V_{\text{fer}} = -\frac{2\pi i}{k} \text{Tr} \left[X_A^\dagger X^A \bar{\Psi}^B \Psi_B + X^A X_A^\dagger \Psi_B \bar{\Psi}^B - 2X^A X_B^\dagger \Psi_A \bar{\Psi}^B - 2X_A^\dagger X^B \bar{\Psi}^A \Psi_B - \epsilon^{ABCD} X_A^\dagger \Psi_B X_C^\dagger \Psi_D + \epsilon_{ABCD} X^A \Psi^{B\dagger} X^C \Psi^{D\dagger} \right]. \quad (2.3)$$

The original ABJM model possesses an $SU(4)_R$ symmetry, under which a field with the upper index A (X^A and $\Psi^{\dagger A}$) transforms as 4 and one with the lower index A (X_A^\dagger and Ψ_A) transforms as $\bar{4}$. The gauge group is $U(N) \times U(N)$ and X^A and Ψ_A transform as (N, \bar{N}) and X_A^\dagger and $\Psi^{\dagger A}$ transform as (\bar{N}, N) . The model is parametrized by one integer k given by the level of the Chern-Simons action.

The ABJM action is invariant under the $\mathcal{N} = 6$ SUSY transformation [28, 29]

$$\begin{aligned} \delta X^A &= \bar{\epsilon}_i (\Gamma^{i*})^{AB} \Psi_B, & \delta X_A^\dagger &= -\bar{\Psi}^{\dagger B} \epsilon_i \Gamma_{AB}^i, \\ \delta \Psi_A &= -i \gamma^\mu \epsilon_i \Gamma_{AB}^i D_\mu X^B \\ &\quad + \frac{2\pi}{k} (-i \epsilon_i \Gamma_{AB}^i (X^C X_C^\dagger X^B - X^B X_C^\dagger X^C) + 2i \epsilon_i \Gamma_{CD} X^C X_A^\dagger X^D), \\ \delta \bar{\Psi}^A &= -i D^\mu X_B^\dagger \bar{\epsilon}_i \gamma_\mu (\Gamma^{i*})^{AB}, \\ &\quad + \frac{2\pi}{k} (i (X_B^\dagger X^C X_C^\dagger - X_C^\dagger X^C X_B^\dagger) \bar{\epsilon}_i (\Gamma^{i*})^{AB} - 2i X_D^\dagger X^A X_C^\dagger \bar{\epsilon}_i (\Gamma^{i*})^{CD}), \\ \delta A_\mu &= -\frac{2\pi}{k} (i X^A \bar{\Psi}^B \gamma_\mu \epsilon_i \Gamma_{AB}^i + i \bar{\epsilon}_i (\Gamma^{i*})^{AB} \gamma_\mu \Psi_A X_B^\dagger), \\ \delta \hat{A}_\mu &= \frac{2\pi}{k} (i \bar{\Psi}^A X^B \gamma_\mu \epsilon_i \Gamma_{AB}^i + i \bar{\epsilon}_i (\Gamma^{i*})^{AB} \gamma_\mu X_A^\dagger \Psi_B), \end{aligned} \quad (2.4)$$

where ϵ_i (for $i = 1, \dots, 6$) are six independent Majorana fermions. We take the explicit form of gamma matrices Γ_{AB}^i as

$$\begin{aligned} \Gamma^1 &= \sigma_2 \otimes 1, & \Gamma^2 &= -i\sigma_2 \otimes \sigma_3, & \Gamma^3 &= i\sigma_2 \otimes \sigma_1, \\ \Gamma^4 &= -\sigma_1 \otimes \sigma_2, & \Gamma^5 &= \sigma_3 \otimes \sigma_2, & \Gamma^6 &= -i1 \otimes \sigma_2. \end{aligned} \tag{2.5}$$

These chiral SO(6) gamma matrices are the intertwiner between the SU(4) antisymmetric representation (with the reality condition) and the SO(6) (real) vector representation. Note that $\frac{1}{2}\epsilon^{ABCD}\Gamma_{CD}^i = -(\Gamma^{i*})^{AB}$. The model is also invariant under the conformal transformation, so that the theory has 12 additional superconformal charges [30].

The mass deformation of the ABJM model was studied in [25–27]. We focus on the maximally supersymmetric mass deformation,

$$\begin{aligned} V_{\text{mass}} &= m^2 \text{Tr}(X_A^\dagger X^A) + im \text{Tr}(\bar{\Psi}^a \Psi_a - \bar{\Psi}^{a'} \Psi_{a'}) \\ &\quad - \frac{4\pi}{k} m \text{Tr}(X^a X_{[a}^\dagger X^b X_{b]}^\dagger - X^{a'} X_{[a'}^\dagger X^{b'} X_{b']}^\dagger), \end{aligned} \tag{2.6}$$

which breaks the SU(4) R-symmetry down to the SU(2) × SU(2) × U(1). We set $A = (a, a')$, where a and a' are two SU(2) indices, and we have introduced the following notation

$$X_{[a} X_{b]} \equiv X_a X_b - X_b X_a .$$

Though the mass term breaks the SU(4) R-symmetry down to the SU(2) × SU(2) × U(1), the $\mathcal{N} = 6$ SUSY remains once we add the SUSY transformation

$$\begin{aligned} \delta_m \Psi_a &= im\epsilon_i \Gamma_{aB}^i X^B, & \delta_m \bar{\Psi}^a &= -im\bar{\epsilon}_i (\Gamma^{i*})^{aB} X_B^\dagger, \\ \delta_m \Psi_{a'} &= -im\epsilon_i \Gamma_{a'B}^i X^B, & \delta_m \bar{\Psi}^{a'} &= im\bar{\epsilon}_i (\Gamma^{i*})^{a'B} X_B^\dagger. \end{aligned} \tag{2.7}$$

The mass deformation obviously breaks the conformal invariance, so the 12 superconformal generators are lost accordingly. From the M-theory viewpoint, the mass deformation corresponds to turning on a background 4-form flux in the bulk. After taking the mass deformation, the theory has multiple vacua including the broken (Higgs) phase, but we focus on the unbroken phase in the following non-relativistic limit analysis.

3 Non-relativistic limit

There are several possible ways to take a non-relativistic limit of the relativistic action. We first investigate the non-relativistic limit which preserves the maximal SUSY. It turns out that the non-relativistic limit preserves 14 supercharges (including 2 superconformal charges).

3.1 Action

We begin with the bosonic part. The relativistic scalar field X^A can be decomposed into two non-relativistic scalar fields ϕ^A and $\hat{\phi}^{*A}$ [15, 21] as

$$X^A = \frac{1}{\sqrt{2m}} \left(e^{-imt} \phi^A + e^{imt} \hat{\phi}^{*A} \right), \tag{3.1}$$

where ϕ^A describes a particle degree of freedom and $\hat{\phi}_A$ describes an anti-particle degree of freedom. To obtain the maximal SUSY transformation, we discard $\hat{\phi}_A$ and only keep ϕ^A .³ After the substitution of our ansatz (3.1), the kinetic part of the original relativistic action is replaced by the Schrödinger action:

$$i\text{Tr}(\phi_A^\dagger D_0 \phi^A) - \frac{1}{2m} \text{Tr}(D_i \phi_A^\dagger D_i \phi^A). \quad (3.2)$$

Similarly, the relativistic fermion field Ψ^A can be decomposed into non-relativistic two-component spinor fields $\psi_{\alpha A}$ and $\hat{\psi}_\alpha^A$ in the following form:

$$\Psi_A = e^{-imt} \psi_A + e^{imt} \sigma_2 \hat{\psi}_A^*. \quad (3.3)$$

Again, in order to obtain the maximal SUSY theory, we discard the anti-particle degrees of freedom $\hat{\psi}_A$. Actually, only the half of the spinor components are dynamical in the non-relativistic limit. To see this, we note that the Dirac equation

$$\begin{pmatrix} iD_0 + m & D_+ \\ D_- & -iD_0 + m \end{pmatrix} \begin{pmatrix} \Psi_{1a} \\ \Psi_{2a} \end{pmatrix} = 0 \quad (D_\pm \equiv D_1 \pm iD_2), \quad (3.4)$$

is decomposed into the two equations:

$$2m\psi_{1a} + D_+\psi_{2a} = 0, \quad D_-\psi_{1a} - iD_0\psi_{2a} = 0, \quad (3.5)$$

in the non-relativistic limit. We can replace the first component of the non-relativistic spinor ψ_{1a} by $-\frac{D_+}{2m}\psi_{2a}$. Then, the non-relativistic equation for the second component of the fermion is given by the Pauli equation:

$$iD_0\psi_{2a} = -\frac{D_-D_+}{2m}\psi_{2a}. \quad (3.6)$$

In the same way, the Dirac equation for $\Psi_{a'}$ is given by

$$\begin{pmatrix} iD_0 - m & D_+ \\ D_- & -iD_0 - m \end{pmatrix} \begin{pmatrix} \Psi_{1a'} \\ \Psi_{2a'} \end{pmatrix} = 0, \quad (3.7)$$

and in the non-relativistic limit, it becomes

$$iD_0\psi_{1a'} + D_+\psi_{2a'} = 0, \quad D_-\psi_{1a'} - 2m\psi_{2a'} = 0. \quad (3.8)$$

We can replace the second component of the non-relativistic spinor $\psi_{2a'}$ by $\frac{D_-}{2m}\psi_{1a'}$, and the first equation yields the Pauli equation:

$$iD_0\psi_{1a'} = -\frac{D_+D_-}{2m}\psi_{1a'}. \quad (3.9)$$

In the following, we drop the subscript 1 (for $\psi_{a'}$) and 2 (for ψ_a) with the above substitution implicitly assumed.

³In the later section, we will investigate other choices of the non-relativistic limit to obtain less super-symmetric theories. We discuss the consistency of the truncation in appendix B.

We now present the non-relativistic ABJM action obtained by substituting the above non-relativistic ansatz. We only keep the quartic potential terms and neglect the sextic terms that are irrelevant deformations in the non-relativistic superconformal limit [15, 21].

Due to the topological nature, there is no change in the Chern-Simons term:

$$S_{\text{CS}} = \frac{k}{4\pi} \int dt d^2x \epsilon^{\mu\nu\lambda} \text{Tr} \left[A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda - \hat{A}_\mu \partial_\nu \hat{A}_\lambda - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda \right]. \quad (3.10)$$

The kinetic terms for bosons and fermions are given by

$$S_{\text{kin}} = \int dt d^2x \left[i \text{Tr}(\phi_A^\dagger D_0 \phi^A) - \frac{1}{2m} \text{Tr}(D_i \phi_A^\dagger D_i \phi^A) + i \text{Tr}(\psi^{\dagger A} D_0 \psi_A) + \frac{1}{2m} \text{Tr}(\psi^{\dagger a} D_- D_+ \psi_a + \psi^{\dagger a'} D_+ D_- \psi_{a'}) \right]. \quad (3.11)$$

We can also rewrite the Pauli terms as

$$\begin{aligned} & \frac{1}{2m} \text{Tr}(\psi^{\dagger a} D_- D_+ \psi_a + \psi^{\dagger a'} D_+ D_- \psi_{a'}) \\ &= \frac{1}{2m} \text{Tr} \left[\psi^{\dagger a} \left(D_i^2 \psi_a - F_{12} \psi_a + \psi_a \hat{F}_{12} \right) \right] + \frac{1}{2m} \text{Tr} \left[\psi^{\dagger a'} \left(D_i^2 \psi_{a'} + F_{12} \psi_{a'} - \psi_{a'} \hat{F}_{12} \right) \right]. \end{aligned}$$

The non-relativistic fields ϕ^a , $\phi^{a'}$, ψ_a and $\psi_{a'}$ all transform as (N, \bar{N}) under $U(N) \times U(N)$.

Let us move on to the potential part. As we have mentioned, we discard the irrelevant sextic potential and we only keep the marginal quartic terms.⁴ The bosonic potential comes from the supersymmetric completion of the mass term in (2.6), leading to

$$S_{\text{bos}} = \frac{\pi}{km} \int dt d^2x \text{Tr} \left(\phi^a \phi_{[a}^\dagger \phi^b \phi_{b]}^\dagger - \phi^{a'} \phi_{[a'}^\dagger \phi^{b'} \phi_{b']}^\dagger \right). \quad (3.12)$$

The fermionic potential comes from the non-relativistic limit of (2.3):

$$\begin{aligned} S_{\text{fer}} = \frac{\pi}{km} \int dt d^2x \text{Tr} & \left[(\phi_a^\dagger \phi^a + \phi_{a'}^\dagger \phi^{a'}) (\psi^{\dagger b} \psi_b - \psi^{\dagger b'} \psi_{b'}) \right. \\ & + (\phi_a^\dagger \phi_a^\dagger + \phi_{a'}^\dagger \phi_{a'}^\dagger) (\psi_b \psi^{\dagger b} - \psi_{b'} \psi^{\dagger b'}) \\ & - 2\phi_a^\dagger \phi_b^\dagger \psi_a \psi^{\dagger b} + 2\phi_{a'}^\dagger \phi_{b'}^\dagger \psi_{a'} \psi^{\dagger b'} - 2\phi_a^\dagger \phi^b \psi^{\dagger a} \psi_b + 2\phi_{a'}^\dagger \phi^{b'} \psi^{\dagger a'} \psi_{b'} \\ & - i\epsilon^{ab} \epsilon^{c'd'} \phi_a^\dagger \psi_b \phi_c^\dagger \psi_{d'} - i\epsilon^{bc} \epsilon^{a'd'} \phi_{a'}^\dagger \psi_b \phi_c^\dagger \psi_{d'} \\ & + i\epsilon^{a'b'} \epsilon^{cd} \phi_{a'}^\dagger \psi_{b'} \phi_c^\dagger \psi_d + i\epsilon^{b'c'} \epsilon^{ad} \phi_a^\dagger \psi_{b'} \phi_c^\dagger \psi_d \\ & + i\epsilon_{ab} \epsilon_{c'd'} \phi^a \psi^{\dagger b} \phi^{c'} \psi^{\dagger d'} + i\epsilon_{bc} \epsilon_{a'd'} \phi^{a'} \psi^{\dagger b} \phi^{c'} \psi^{\dagger d'} \\ & \left. - i\epsilon_{a'b'} \epsilon_{cd} \phi^{a'} \psi^{\dagger b'} \phi^c \psi^{\dagger d} - i\epsilon_{b'c'} \epsilon_{ad} \phi^a \psi^{\dagger b'} \phi^{c'} \psi^{\dagger d} \right]. \quad (3.13) \end{aligned}$$

Here, we have dropped the higher dimensional terms including the derivatives of fermions. The final non-relativistic ABJM action is given by the sum of (3.10), (3.11), (3.12) and (3.13).

⁴Note the classical scaling dimension of the non-relativistic fields $D(\phi^a) = D(\phi^{a'}) = D(\psi_a) = D(\psi_{a'}) = 1$.

3.2 Bosonic symmetry

Let us investigate the symmetry of the non-relativistic ABJM model. First of all, the model is invariant under the bosonic Schrödinger symmetry (+ some internal symmetries):

- time translation: $\delta t = -a$

$$\begin{aligned} \delta\phi^A &= aD_0\phi^A, & \delta\psi_A &= aD_0\psi_A, \\ \delta A_0 &= \delta\hat{A}_0 = 0, & \delta A_i &= aF_{0i}, & \delta\hat{A}_i &= a\hat{F}_{0i}, \end{aligned} \quad (3.14)$$

with the conserved charge (Hamiltonian)

$$\begin{aligned} H = \int d^2x \left[\frac{1}{2m} \text{Tr}(D_i\phi_A^\dagger D_i\phi^A + D_i\psi^{\dagger A} D_i\psi_A) + \frac{1}{2m} \text{Tr}(\psi^{\dagger a}(F_{12}\psi_a - \psi_a\hat{F}_{12})) \right. \\ \left. - \frac{1}{2m} \text{Tr}(\psi^{\dagger a'}(F_{12}\psi_{a'} - \psi_{a'}\hat{F}_{12})) + V_{\text{bos}} + V_{\text{fer}} \right]. \end{aligned} \quad (3.15)$$

- spatial translation: $\delta x^i = a^i$

$$\begin{aligned} \delta\phi^A &= -a^i D_i\phi^A, & \delta\psi_A &= -a^i D_i\psi_A, \\ \delta A_0 &= a^i F_{0i}, & \delta\hat{A}_0 &= a^i \hat{F}_{0i}, & \delta A_i &= \epsilon_{ij} a^j F_{12}, & \delta\hat{A}_i &= \epsilon_{ij} a^j \hat{F}_{12}, \end{aligned} \quad (3.16)$$

with the conserved charge (momentum)

$$P_i = \int d^2x p_i, \quad p_i = -\frac{i}{2} \text{Tr} \left[\phi_A^\dagger D_i\phi^A - D_i\phi_A^\dagger \phi^A + \psi^{\dagger A} D_i\psi_A - D_i\psi^{\dagger A} \psi_A \right]. \quad (3.17)$$

- infinitesimal rotation: $\delta x_i = -\theta\epsilon_{ij}x^j$

$$\begin{aligned} \delta\phi^A &= \theta\epsilon_{ij}x^i D^j\phi^A, & \delta\psi_a &= \theta\epsilon_{ij}x^i D^j\psi_a + \frac{i}{2}\theta\psi_a, & \delta\psi_{a'} &= \theta\epsilon_{ij}x^i D^j\psi_{a'} - \frac{i}{2}\theta\psi_{a'}, \\ \delta A_0 &= -\theta\epsilon_{ij}x^i F_{0j}, & \delta\hat{A}_0 &= -\theta\epsilon_{ij}x^i \hat{F}_{0j}, & \delta A_i &= -\theta x_i F_{12}, & \delta\hat{A}_i &= -\theta x_i \hat{F}_{12}, \end{aligned} \quad (3.18)$$

with the conserved charge (U(1) angular momentum)⁵

$$J = - \int d^2x \left[\epsilon^{ij} x_i p_j + \frac{1}{2} \text{Tr}(\psi^{\dagger a} \psi_a - \psi^{\dagger a'} \psi_{a'}) \right]. \quad (3.19)$$

- total number density (actually a part of the gauge symmetry)

$$\delta\phi^A = -i\alpha m\phi^A, \quad \delta\psi_A = -i\alpha m\psi_A, \quad \delta A_\mu = \delta\hat{A}_\mu = 0, \quad (3.20)$$

with the conserved charge (total mass operator)

$$M = m \int d^2x \rho, \quad \rho = \text{Tr}(\phi_A^\dagger \phi^A + \psi^{\dagger A} \psi_A). \quad (3.21)$$

⁵We have added the separately conserved U(1)_F to obtain the conventional spin 1/2 of fermions. Actually, in two spatial-dimension, the addition of arbitrary amount of U(1)_F (or U(1)_B) does not change the Schrödinger algebra.

- infinitesimal Galilean boost: $\delta x^i = -v^i t$

$$\begin{aligned}
 \delta\phi^A &= -(imv^i x_i - tv^i D_i)\phi^A, & \delta\psi_A &= -(imv^i x_i - tv^i D_i)\psi_A, \\
 \delta A_0 &= -tv^i F_{0i}, & \delta\hat{A}_0 &= -tv^i \hat{F}_{0i}, \\
 \delta A_i &= -t\epsilon_{ij}v^j F_{12}, & \delta\hat{A}_i &= -t\epsilon_{ij}v^j \hat{F}_{12},
 \end{aligned} \tag{3.22}$$

with the conserved charge

$$G_i = \int d^2x [-tp_i + mx_i\rho]. \tag{3.23}$$

- infinitesimal dilatation: $\delta t = 2\alpha t$, $\delta x^i = \alpha x^i$

$$\begin{aligned}
 \delta\phi^A &= -\alpha(1 + x^i D_i + 2tD_0)\phi^A, & \delta\psi_A &= -\alpha(1 + x^i D_i + 2tD_0)\psi_A, \\
 \delta A_0 &= \alpha x^i F_{0i}, & \delta\hat{A}_0 &= \alpha x^i \hat{F}_{0i}, \\
 \delta A_i &= \alpha(\epsilon_{ij}x^j F_{12} - 2tF_{0i}), & \delta\hat{A}_i &= \alpha(\epsilon_{ij}x^j \hat{F}_{12} - 2t\hat{F}_{0i}),
 \end{aligned} \tag{3.24}$$

with the conserved charge

$$D = -2tH + \int d^2x x^i p_i. \tag{3.25}$$

- infinitesimal special conformal transformation: $\delta t = -at^2$, $\delta x^i = -atx^i$

$$\begin{aligned}
 \delta\phi^A &= \left(at - \frac{i}{2}max^2 + atx^i D_i + at^2 D_0\right)\phi^A \\
 \delta\psi_A &= \left(at - \frac{i}{2}max^2 + atx^i D_i + at^2 D_0\right)\psi_A \\
 \delta A_0 &= -atx^i F_{0i}, & \delta\hat{A}_0 &= -atx^i \hat{F}_{0i} \\
 \delta A_i &= -at\epsilon_{ij}x^j F_{12} + at^2 F_{0i}, & \delta\hat{A}_i &= -at\epsilon_{ij}x^j \hat{F}_{12} + at^2 \hat{F}_{0i},
 \end{aligned} \tag{3.26}$$

with the conserved charge

$$K = -t^2 H - tD + \frac{1}{2}m \int d^2x x^2 \rho. \tag{3.27}$$

These generators satisfy the Schrödinger algebra⁶

$$\begin{aligned}
 i[J, P_i] &= \epsilon_{ij}P_j, & i[J, G_i] &= \epsilon_{ij}G_j, & i[P_i, G_j] &= \delta_{ij}M, & i[H, G_i] &= P_i, \\
 i[D, H] &= -2H, & i[D, K] &= 2K, & i[H, K] &= D, & i[K, P_i] &= -G_i, \\
 i[D, G_i] &= G_i, & i[D, P_i] &= -P_i.
 \end{aligned} \tag{3.28}$$

To derive these, as in [15], it is useful to note that A_i and \hat{A}_i are solved by $A_+ = \hat{A}_+ = 0$, $A_- = -\frac{4\pi}{k}i\partial_- \int d^2y G(x-y)(\phi^A \phi_A^\dagger - \psi_A \psi^{\dagger A})(y)$ and $\hat{A}_- = -\frac{4\pi}{k}i\partial_- \int d^2y G(x-y)(\phi_A^\dagger \phi^A + \psi^{\dagger A} \psi_A)(y)$ where $G(x-y) = \frac{1}{2\pi} \log|x-y|$.

In addition, the model possesses some internal global symmetries:

⁶The Poisson bracket (more precisely Dirac bracket) is defined by $[F, G]_{PB} = -i \left(-\frac{\delta F}{\delta\phi^*} \frac{\delta G}{\delta\phi} + \frac{\delta F}{\delta\phi} \frac{\delta G}{\delta\phi^*} \right) - i \left(\frac{\delta^r F}{\delta\psi^*} \frac{\delta^l G}{\delta\psi} + \frac{\delta^r F}{\delta\psi} \frac{\delta^l G}{\delta\psi^*} \right)$, where $\frac{\delta^r}{\delta\psi}$ denotes the right derivative and $\frac{\delta^l}{\delta\psi}$ denotes the left derivative. We further replace the Poisson bracket with the quantum mechanical (anti-)commutator $[F, G]_{PB} \rightarrow -i[F, G]$ or $-i\{F, G\}$.

- $U(1)_B \times U(1)_F$

$$\begin{aligned} \delta\phi^a &= -i\alpha\phi^a, & \delta\phi^{a'} &= i\alpha\phi^{a'}, & \delta\psi_a &= -i\beta\psi_a, & \delta\psi_{a'} &= i\beta\psi_{a'}, \\ \delta A_\mu &= \delta\hat{A}_\mu = 0, \end{aligned} \tag{3.29}$$

with the conserved charges

$$Q_B = \int d^2x \operatorname{Tr}(\phi_a^\dagger\phi^a - \phi_{a'}^\dagger\phi^{a'}), \quad Q_F = \int d^2x \operatorname{Tr}(\psi^{\dagger a}\psi_a - \psi^{\dagger a'}\psi_{a'}). \tag{3.30}$$

We have used Q_F to improve the $U(1)$ angular momentum. The diagonal part $\alpha = \beta$ is a part of the gauge symmetry.

- $SU(2) \times SU(2)$ R-symmetry

The first $SU(2)$ is generated by

$$\delta\phi^a = i\alpha^i(\sigma_i)_b^a\phi^b, \quad \delta\psi_a = -i\alpha^i(\sigma_i^*)_a^b\psi_b, \quad \delta(\text{others}) = 0. \tag{3.31}$$

The corresponding generator is

$$R_i^{(1)} = \int d^2x \operatorname{Tr} \left(\phi_a^\dagger(\sigma_i)_b^a\phi^b - \psi^{\dagger a}(\sigma_i^*)_a^b\psi_b \right). \tag{3.32}$$

Similarly,

$$\delta\phi^{a'} = i\alpha^i(\sigma_i)_{b'}^{a'}\phi^{b'}, \quad \delta\psi_{a'} = -i\alpha^i(\sigma_i^*)_{a'}^{b'}\psi_{b'}, \quad \delta(\text{others}) = 0. \tag{3.33}$$

The corresponding generator is

$$R_i^{(2)} = \int d^2x \operatorname{Tr} \left(\phi_{a'}^\dagger(\sigma_i)_{b'}^{a'}\phi^{b'} - \psi^{\dagger a'}(\sigma_i^*)_{a'}^{b'}\psi_{b'} \right). \tag{3.34}$$

The above global internal symmetries commute with all the bosonic generators of the Schrödinger algebra.

3.3 Supersymmetry

The non-relativistic limit of the mass deformed ABJM model has the non-relativistic supersymmetry induced from the supersymmetry of the original relativistic theory. Let us first begin with the kinematical SUSY. The first order supersymmetry is obtained by the direct non-relativistic limit of the relativistic supersymmetry. They are generated by the following charges

$$\begin{aligned} \epsilon\mathcal{Q} &= i\sqrt{2m} \left[(\epsilon_1 + i\epsilon_2)\operatorname{Tr}(i\phi_1^\dagger\psi_2 - i\phi_2^\dagger\psi_1 - \phi^{1'}\psi^{\dagger 2'} + \phi^{2'}\psi^{\dagger 1'}) \right. \\ &\quad + \epsilon_3\operatorname{Tr}(-i\phi^1\psi^{\dagger 2'} + i\phi^2\psi^{\dagger 1'} + \phi_1^\dagger\psi_2 - \phi_2^\dagger\psi_1) \\ &\quad + \epsilon_4\operatorname{Tr}(\phi^1\psi^{\dagger 2'} + \phi^2\psi^{\dagger 1'} + i\phi_1^\dagger\psi_2 + i\phi_2^\dagger\psi_1) \\ &\quad + \epsilon_5\operatorname{Tr}(-\phi^1\psi^{\dagger 1'} + \phi^2\psi^{\dagger 2'} - i\phi_1^\dagger\psi_1 + i\phi_2^\dagger\psi_2) \\ &\quad \left. + \epsilon_6(i\phi^1\psi^{\dagger 1'} + i\phi^2\psi^{\dagger 2'} + \phi_1^\dagger\psi_1 + \phi_2^\dagger\psi_2) \right], \end{aligned} \tag{3.35}$$

and similarly by $\epsilon^* Q^*$ by just complex conjugation. There are total five independent complex supercharges,⁷ and we relabel them so that

$$Q_1^{\hat{i}} \equiv \sqrt{2m} \int d^2x j_{\hat{i}} \quad (\hat{i} = 0, 3 \dots, 6), \quad (3.36)$$

where

$$\begin{aligned} j_0 &= \text{Tr}(i\phi_1^\dagger \psi_2 - i\phi_2^\dagger \psi_1 - \phi^{1'} \psi^{\dagger 2'} + \phi^{2'} \psi^{\dagger 1'}), \\ j_3 &= \text{Tr}(-i\phi^1 \psi^{\dagger 2'} + i\phi^2 \psi^{\dagger 1'} + \phi_1^\dagger \psi_2 - \phi_2^\dagger \psi_1), \\ j_4 &= \text{Tr}(\phi^1 \psi^{\dagger 2'} + \phi^2 \psi^{\dagger 1'} + i\phi_1^\dagger \psi_2 + i\phi_2^\dagger \psi_1), \\ j_5 &= \text{Tr}(-\phi^1 \psi^{\dagger 1'} + \phi^2 \psi^{\dagger 2'} - i\phi_1^\dagger \psi_1 + i\phi_2^\dagger \psi_2), \\ j_6 &= \text{Tr}(i\phi^1 \psi^{\dagger 1'} + i\phi^2 \psi^{\dagger 2'} + \phi_1^\dagger \psi_1 + \phi_2^\dagger \psi_2). \end{aligned} \quad (3.37)$$

Q_1^0 is singlet under the $SU(2) \times SU(2)$ R-symmetry while $Q_1^{\hat{i}}$ ($\hat{i} = 3, \dots, 6$) transform as 2×2 representations under the $SU(2) \times SU(2)$.

We can compute the anti-commutation relations as

$$\begin{aligned} \{Q_1^0, Q_1^{0*}\} &= 2M, & \{Q_1^{m*}, Q_1^n\} &= 2M\delta^{mn} - 2imR^{mn}, \\ \{Q_1^0, Q_1^{m*}\} &= \{Q_1^{\hat{i}}, Q_1^{\hat{j}}\} = 0, \\ i[J, Q_1^0] &= \frac{i}{2}Q_1^0, & i[J, Q_1^m] &= \frac{i}{2}Q_1^m, \\ [H, Q_1^{\hat{i}}] &= [P_i, Q_1^{\hat{i}}] = [G_i, Q_1^{\hat{i}}] = [D, Q_1^{\hat{i}}] = [K, Q_1^{\hat{i}}] = [M, Q_1^{\hat{i}}] = 0. \end{aligned} \quad (3.38)$$

R^{mn} are particular combinations of the $SU(2) \times SU(2)$ R-charges introduced in (3.34):

$$\begin{aligned} R^{34} &= \int d^2x \text{Tr}(-\psi^{\dagger 2'} \psi_{2'} + \psi^{\dagger 1'} \psi_{1'} + \psi_2 \psi^{\dagger 2} - \psi_1 \psi^{\dagger 1} - \phi^1 \phi_1^\dagger + \phi^2 \phi_2^\dagger - \phi_1^\dagger \phi^{1'} + \phi_2^\dagger \phi^{2'}), \\ R^{35} &= \int d^2x \text{Tr}(\psi^{\dagger 2'} \psi_{1'} + \psi^{\dagger 1'} \psi_{2'} - \psi_2 \psi^{\dagger 1} - \psi_1 \psi^{\dagger 2} - \phi^1 \phi_2^\dagger - \phi^2 \phi_1^\dagger - \phi_1^\dagger \phi^{2'} - \phi_2^\dagger \phi^{1'}), \\ R^{36} &= \int d^2x \text{Tr}(i\psi^{\dagger 2'} \psi_{1'} - i\psi^{\dagger 1'} \psi_{2'} + i\psi_2 \psi^{\dagger 1} - i\psi_1 \psi^{\dagger 2} + i\phi^1 \phi_2^\dagger - i\phi^2 \phi_1^\dagger - i\phi_1^\dagger \phi^{2'} + i\phi_2^\dagger \phi^{1'}), \\ R^{45} &= \int d^2x \text{Tr}(i\psi^{\dagger 2'} \psi_{1'} - i\psi^{\dagger 1'} \psi_{2'} - i\psi_2 \psi^{\dagger 1} + i\psi_1 \psi^{\dagger 2} - i\phi^1 \phi_2^\dagger + i\phi^2 \phi_1^\dagger - i\phi_1^\dagger \phi^{2'} + i\phi_2^\dagger \phi^{1'}), \\ R^{46} &= \int d^2x \text{Tr}(-\psi^{\dagger 2'} \psi_{1'} - \psi^{\dagger 1'} \psi_{2'} - \psi_2 \psi^{\dagger 1} - \psi_1 \psi^{\dagger 2} - \phi^1 \phi_2^\dagger - \phi^2 \phi_1^\dagger + \phi_1^\dagger \phi^{2'} + \phi_2^\dagger \phi^{1'}), \\ R^{56} &= \int d^2x \text{Tr}(-\psi^{\dagger 2'} \psi_{2'} + \psi^{\dagger 1'} \psi_{1'} - \psi_2 \psi^{\dagger 2} + \psi_1 \psi^{\dagger 1} + \phi^1 \phi_1^\dagger - \phi^2 \phi_2^\dagger - \phi_1^\dagger \phi^{1'} + \phi_2^\dagger \phi^{2'}). \end{aligned} \quad (3.39)$$

Since the particular combination of the SUSY parameter $\epsilon_1 + i\epsilon_3$ does not generate the first order kinematical SUSY transformation, one can construct the second dynamical SUSY transformation [21, 23]. The second SUSY is generated by the supercharge

$$Q_2 = \frac{1}{\sqrt{2m}} \int d^2x \text{Tr} \left(\phi_1^\dagger D_+ \psi_2 - \phi_2^\dagger D_+ \psi_1 - i\phi^{1'} D_+ \psi^{\dagger 2'} + i\phi^{2'} D_+ \psi^{\dagger 1'} \right). \quad (3.40)$$

⁷Note that $\epsilon_1 - i\epsilon_2$ does not appear in the first supercharges, which results in the emergence of the second dynamical SUSY.

The supercharge Q_2 is invariant under $SU(2) \times SU(2)$ R-symmetry. The anti-commutation relations for Q_2 can be computed as

$$\begin{aligned}
 \{Q_2, Q_2^*\} &= H, & \{Q_1^0, Q_2^*\} &= P_-, & \{Q_1^0, Q_2\} &= \{Q_1^m, Q_2^*\} = \{Q_1^m, Q_2\} = 0, \\
 [P_i, Q_2] &= [H, Q_2] = 0, & i[J, Q_2] &= -\frac{i}{2}Q_2, \\
 i[G_+, Q_2^*] &= -Q_1^{0*}, & i[G_-, Q_2] &= -Q_1^0, & i[D, Q_2] &= -Q_2, \\
 i[K, Q_2] &= tQ_2 - \sqrt{\frac{m}{2}} \int d^2x x^+ j_0, & [M, Q_2] &= [R^{mn}, Q_2] = 0.
 \end{aligned} \tag{3.41}$$

As expected from the first anti-commutation relation in (3.41), we can rewrite the Hamiltonian (3.15) by using the Gauss law constraints

$$F_{12} = \frac{2\pi}{k} (\phi^A \phi_A^\dagger - \psi_A \psi^{\dagger A}), \quad \hat{F}_{12} = \frac{2\pi}{k} (\phi_A^\dagger \phi^A + \psi^{\dagger A} \psi_A), \tag{3.42}$$

in a manifestly semi-positive definite form:

$$\begin{aligned}
 H &= \int d^2x \left[\frac{1}{2m} \text{Tr}((D_- \phi^a)^\dagger D_- \phi^a + (D_+ \phi^{a'})^\dagger D_+ \phi^{a'}) \right. \\
 &\quad + \frac{1}{2m} \text{Tr}((D_+ \psi_a)^\dagger D_+ \psi_a + (D_- \psi_{a'})^\dagger D_- \psi_{a'}) \\
 &\quad + \frac{2\pi}{mk} \text{Tr}((\epsilon_{ab} \phi^a \psi^{\dagger b} - i\epsilon^{a'b'} \psi_{b'} \phi_a^\dagger)^\dagger (\epsilon_{ab} \phi^a \psi^{\dagger b} - i\epsilon^{a'b'} \psi_{b'} \phi_a^\dagger)) \\
 &\quad \left. + \frac{2\pi}{mk} \text{Tr}((\epsilon^{ab} \phi_a^\dagger \psi_b + i\epsilon_{a'b'} \psi^{\dagger b'} \phi^{a'})^\dagger (\epsilon^{ab} \phi_a^\dagger \psi_b + i\epsilon_{a'b'} \psi^{\dagger b'} \phi^{a'})) \right].
 \end{aligned} \tag{3.43}$$

The commutator of K and Q_2 defines the superconformal charge

$$i[K, Q_2] = S, \tag{3.44}$$

so that

$$S = tQ_2 - \sqrt{\frac{m}{2}} \int d^2x x^+ j_0 \quad (x^\pm \equiv x^1 \pm ix^2). \tag{3.45}$$

Then the anti-commutation relations containing S are

$$\begin{aligned}
 \{Q_1^0, S^*\} &= -G_-, & \{Q_1^{m*}, S\} &= \{Q_1^m, S\} = \{Q_1^0, S\} = 0, \\
 \{Q_2^*, S\} &= -\frac{1}{2}D - \frac{i}{2}J + \frac{3}{4}iR, & \{S, S^*\} &= K,
 \end{aligned} \tag{3.46}$$

where R is an R-symmetry generator defined as

$$R \equiv - \int d^2x \text{Tr} \left(\frac{2}{3} \phi_a^\dagger \phi^a - \frac{2}{3} \phi_{a'}^\dagger \phi^{a'} - \frac{1}{3} \psi^{\dagger a} \psi_a + \frac{1}{3} \psi^{\dagger a'} \psi_{a'} \right). \tag{3.47}$$

In fact, R generates the $U(1)$ R-symmetry

$$\begin{aligned}
 i[R, Q_1^0] &= -iQ_1^0, & i[R, Q_1^m] &= i\frac{1}{3}Q_1^m, \\
 i[R, Q_2] &= -iQ_2, & i[R, S] &= -iS,
 \end{aligned} \tag{3.48}$$

and commutes with all bosonic generators

$$[R, T_B] = 0, \quad T_B = \{P_i, H, J, G_i, D, K, M, R^{mn}, R\}. \quad (3.49)$$

Finally the remaining non-trivial commutation relations are

$$\begin{aligned} i[P_-, S] &= -Q_1^0, & i[H, S] &= -Q_2, & i[J, S] &= -\frac{i}{2}S, \\ i[D, S] &= S, & [G_i, S] &= [K, S] = [M, S] = [R^{mn}, S] &= 0. \end{aligned} \quad (3.50)$$

3.4 Summary of the superconformal algebra

We summarize the superconformal algebra with 14 fermionic generators obtained in this section. The bosonic part is nothing but the Schrödinger algebra:

$$\begin{aligned} i[J, P_+] &= -iP_+, & i[J, P_-] &= iP_-, & i[J, G_+] &= -iG_+, & i[J, G_-] &= iG_-, \\ i[H, G_+] &= P_+, & i[H, G_-] &= P_-, & i[K, P_+] &= -G_+, & i[K, P_-] &= -G_-, \\ i[D, P_+] &= -P_+, & i[D, P_-] &= -P_-, & i[D, G_+] &= G_+, & i[D, G_-] &= G_-, \\ i[D, H] &= -2H, & i[H, K] &= D, & i[D, K] &= 2K, & i[P_+, G_-] &= 2M. \end{aligned} \quad (3.51)$$

The fermionic part is

$$\begin{aligned} \{Q_1^0, Q_1^{0*}\} &= 2M, & \{Q_1^{m*}, Q_1^n\} &= 2M\delta^{mn} - 2miR^{mn}, \\ \{Q_2, Q_2^*\} &= H, & \{Q_1, Q_2^*\} &= P_-, & \{Q_2, Q_1^*\} &= P_+, \\ i[J, Q_1^0] &= \frac{i}{2}Q_1^0, & i[J, Q_1^m] &= \frac{i}{2}Q_1^m, & i[J, Q_2] &= -\frac{i}{2}Q_2, \\ i[G_-, Q_2] &= -Q_1^0, & i[G_+, Q_2^*] &= -Q_1^{0*}, & i[D, Q_2] &= -Q_2, & i[D, Q_2^*] &= -Q_2^*, \\ i[K, Q_2] &= S, & i[H, S^*] &= -Q_2^*, & i[P_-, S] &= -Q_1^0, & i[J, S] &= -\frac{i}{2}S, \\ \{S, S^*\} &= K, & \{S, Q_1^{*0}\} &= -G_+, & i[D, S] &= S, & \{S, Q_2^*\} &= \frac{i}{2}\left(iD - J + \frac{3}{2}R\right), \\ i[R, Q_1^0] &= -iQ_1^0, & i[R, Q_1^m] &= \frac{i}{3}Q_1^m, & i[R, Q_2] &= -iQ_2, & i[R, S] &= -iS. \end{aligned} \quad (3.52)$$

4 Less SUSY limit

In this section, we study other non-relativistic limits of the mass deformed ABJM model, which lead to less supersymmetric theories. The result is summarized in table 1. We only consider the non-relativistic limit which preserves $SU(2) \times SU(2)$ global symmetry while it is possible to obtain less and less SUSY limit by breaking $SU(2) \times SU(2)$ global symmetry.

4.1 8 SUSY limit

Let us take the ansatz for the non-relativistic limit of scalars as

$$X^a = \frac{1}{\sqrt{2m}}e^{-imt}\phi^a, \quad X^{a'} = \frac{1}{\sqrt{2m}}e^{imt}\hat{\phi}^{*a'}, \quad (4.1)$$

| Limit | X^a | $X^{a'}$ | Ψ_a | $\Psi_{a'}$ | Q_1 | Q_2 | S |
|-------|-------|----------|----------|-------------|-------|-------|-----|
| 3 | P | P | P | P | 10 | 2 | 2 |
| 4.1 | P | A | A | P | 8 | 0 | 0 |
| 4.2 | P | A | P | A | 4 | 0 | 0 |
| 4.3 | P | P | A | A | 0 | 0 | 0 |

Table 1. The matter contents of possible non-relativistic limits that preserve $SU(2) \times SU(2)$ and non-trivial supersymmetries. P and A denote particle and anti-particle, respectively.

and fermions as

$$\Psi_a = e^{imt} \sigma_2 \hat{\psi}_a^*, \quad \Psi_{a'} = e^{-imt} \psi_{a'}. \quad (4.2)$$

The Dirac equation for Ψ_a gives slightly different results from those in section 3:

$$\Psi_a = e^{imt} \begin{pmatrix} -i\hat{\psi}_a^* \\ i\frac{D_-}{2m}\hat{\psi}_a^* \end{pmatrix}. \quad (4.3)$$

The action is given by $S_{CS} + S_{kin} + S_{bos} + S_{fer}$, where S_{CS} is the same as in (3.10) while the kinetic term is given by

$$S_{kin} = \int dt d^2x \left[i\text{Tr}(\phi_a^\dagger D_0 \phi^a + \hat{\phi}^{\dagger a'} D_0 \hat{\phi}_{a'}) - \frac{1}{2m} \text{Tr}(D_i \phi_a^\dagger D_i \phi^a + D_i \hat{\phi}^{\dagger a'} D_i \hat{\phi}_{a'}) \right. \\ \left. + i\text{Tr}(\hat{\psi}_a^\dagger D_0 \hat{\psi}^a + \psi^{\dagger a'} D_0 \psi_{a'}) + \frac{1}{2m} \text{Tr}(\hat{\psi}_a^\dagger D_- D_+ \hat{\psi}^a + \psi^{\dagger a'} D_+ D_- \psi_{a'}) \right]. \quad (4.4)$$

Now, ϕ^a and $\psi_{a'}$ transform as (N, \bar{N}) under $U(N) \times U(N)$ whereas $\hat{\phi}_{a'}$ and $\hat{\psi}^a$ transform as (\bar{N}, N) .

The leading bosonic potential that will survive in the conformal limit is

$$S_{bos} = \int dt d^2x \frac{\pi}{km} \text{Tr}(\phi^a \phi_{[a}^\dagger \phi^b \phi_{b]}^\dagger - \hat{\phi}^{\dagger a'} \hat{\phi}_{[a'} \hat{\phi}^{\dagger b'} \hat{\phi}_{b']}). \quad (4.5)$$

The fermionic potential comes from the non-relativistic limit of (2.3):

$$S_{fer} = -\frac{\pi}{km} \int dt d^2x \text{Tr} \left[(\phi_a^\dagger \phi^a + \hat{\phi}_{a'} \hat{\phi}^{\dagger a'}) (\hat{\psi}_b^\dagger \hat{\psi}_b^\dagger + \psi_b^\dagger \psi_b) \right. \\ \left. + (\phi^a \phi_a^\dagger + \hat{\phi}^{\dagger a'} \hat{\phi}_{a'}) (\hat{\psi}_b^\dagger \hat{\psi}_b^\dagger + \psi_b^\dagger \psi_b) \right. \\ \left. - 2(\phi^a \phi_b^\dagger \hat{\psi}_a^\dagger \hat{\psi}_b^\dagger - i\phi^a \hat{\phi}_{b'} \hat{\psi}_a^\dagger \psi_b^\dagger + i\hat{\phi}^{\dagger a'} \phi_b^\dagger \psi_{a'} \hat{\psi}_b^\dagger + \hat{\phi}^{\dagger a'} \hat{\phi}_{b'} \psi_{a'} \psi_b^\dagger) \right. \\ \left. - 2(\phi_a^\dagger \phi^b \hat{\psi}^a \hat{\psi}_b^\dagger + i\phi_a^\dagger \hat{\phi}^{\dagger b'} \hat{\psi}^a \psi_{b'} - i\hat{\phi}_{a'} \phi^b \psi^{\dagger a'} \hat{\psi}_b^\dagger + \hat{\phi}_{a'} \hat{\phi}^{\dagger b'} \psi^{\dagger a'} \psi_b) \right]. \quad (4.6)$$

Let us study the bosonic symmetry of the theory. The theory possesses the full Schrödinger symmetry and $SU(2) \times SU(2)$ R-symmetry acting on indices a and a' . In addition, the theory is invariant under $U(1)_B$ and $U(1)_F$ generated by $Q_B(\phi^a, \hat{\phi}_{a'}, \hat{\psi}^a, \psi_{a'}) = (1, -1, 0, 0)$ and $Q_F(\phi^a, \hat{\phi}_{a'}, \hat{\psi}^a, \psi_{a'}) = (0, 0, 1, -1)$. Furthermore, because ϵ^{ab} and $\epsilon^{a'b'}$

do not appear in the action, the $SU(2) \times SU(2)$ symmetry is enhanced to $U(2) \times U(2)$ with additional $U(1)_R$ charge generated by $Q_{R_1}(\phi^a, \hat{\phi}_{a'}, \hat{\psi}^a, \psi_{a'}) = (1, 0, 1, 0)$ and $Q_{R_2}(\phi^a, \hat{\phi}_{a'}, \hat{\psi}^a, \psi_{a'}) = (0, 1, 0, 1)$.⁸

We now consider the SUSY transformation. The supersymmetries generated by Γ^1 and Γ^2 do not act on the fields non-trivially any longer because the particles cannot transform into anti-particles in the non-relativistic limit. The only non-trivial SUSY transformations are generated by Γ^{3-6} .

The corresponding SUSY generators are

$$\begin{aligned} Q_1^3 &= \sqrt{2mi} \int d^2x \text{Tr} \left(-i\phi^1 \psi^{\dagger 2'} + i\phi^2 \psi^{\dagger 1'} - \hat{\phi}^{\dagger 1'} \hat{\psi}^{\dagger 2} + \hat{\phi}^{\dagger 2'} \hat{\psi}^{\dagger 1} \right), \\ Q_1^4 &= \sqrt{2mi} \int d^2x \text{Tr} \left(\phi^1 \psi^{\dagger 2'} + \phi^2 \psi^{\dagger 1'} + i\hat{\phi}^{\dagger 1'} \hat{\psi}^2 + i\hat{\phi}^{\dagger 2'} \hat{\psi}^1 \right), \\ Q_1^5 &= \sqrt{2mi} \int d^2x \text{Tr} \left(-\phi^1 \psi^{\dagger 1'} + \phi^2 \psi^{\dagger 2'} - i\hat{\phi}^{\dagger 1'} \hat{\psi}^1 + i\hat{\phi}^{\dagger 2'} \hat{\psi}^2 \right), \\ Q_1^6 &= \sqrt{2mi} \int d^2x \text{Tr} \left(i\phi^1 \psi^{\dagger 1'} + i\phi^2 \psi^{\dagger 2'} - \hat{\phi}^{\dagger 1'} \hat{\psi}^1 - \hat{\phi}^{\dagger 2'} \hat{\psi}^2 \right). \end{aligned} \quad (4.7)$$

We can compute the anti-commutation relations as

$$\begin{aligned} \{Q_1^{m*}, Q_1^n\} &= 2M\delta^{mn} - 2miR^{mn}, \quad i[J, Q_1^m] = \frac{i}{2}Q_1^m, \\ [H, Q_1^n] &= [P_i, Q_1^n] = [G_i, Q_1^n] = [D, Q_1^n] = [K, Q_1^n] = [M, Q_1^n] = 0. \end{aligned} \quad (4.8)$$

R^{mn} are particular combinations of the $SU(2) \times SU(2)$ R-charges:

$$\begin{aligned} R^{34} &= \int d^2x \text{Tr}(-\psi^{\dagger 2'} \psi_{2'} + \psi^{\dagger 1'} \psi_{1'} - \hat{\psi}^2 \hat{\psi}_2^\dagger + \hat{\psi}^1 \hat{\psi}_1^\dagger - \phi^1 \phi_1^\dagger + \phi^2 \phi_2^\dagger + \hat{\phi}^{\dagger 1'} \hat{\phi}_{1'} - \hat{\phi}^{\dagger 2'} \hat{\phi}_{2'}), \\ R^{35} &= \int d^2x \text{Tr}(\psi^{\dagger 2'} \psi_{1'} + \psi^{\dagger 1'} \psi_{2'} + \hat{\psi}^2 \hat{\psi}_1^\dagger + \hat{\psi}^1 \hat{\psi}_2^\dagger - \phi^1 \phi_2^\dagger - \phi^2 \phi_1^\dagger + \hat{\phi}^{\dagger 1'} \hat{\phi}_{2'} + \hat{\phi}^{\dagger 2'} \hat{\phi}_{1'}), \\ R^{36} &= \int d^2x \text{Tr}(i\psi^{\dagger 2'} \psi_{1'} - i\psi^{\dagger 1'} \psi_{2'} + i\hat{\psi}^2 \hat{\psi}_1^\dagger - i\hat{\psi}^1 \hat{\psi}_2^\dagger + i\phi^1 \phi_2^\dagger - i\phi^2 \phi_1^\dagger - i\hat{\phi}^{\dagger 1'} \hat{\phi}_{2'} + i\hat{\phi}^{\dagger 2'} \hat{\phi}_{1'}), \\ R^{45} &= \int d^2x \text{Tr}(i\psi^{\dagger 2'} \psi_{1'} - i\psi^{\dagger 1'} \psi_{2'} - i\hat{\psi}^2 \hat{\psi}_1^\dagger + i\hat{\psi}^1 \hat{\psi}_2^\dagger - i\phi^1 \phi_2^\dagger + i\phi^2 \phi_1^\dagger - i\hat{\phi}^{\dagger 1'} \hat{\phi}_{2'} + i\hat{\phi}^{\dagger 2'} \hat{\phi}_{1'}), \\ R^{46} &= \int d^2x \text{Tr}(-\psi^{\dagger 2'} \psi_{1'} - \psi^{\dagger 1'} \psi_{2'} + \hat{\psi}^2 \hat{\psi}_1^\dagger + \hat{\psi}^1 \hat{\psi}_2^\dagger - \phi^1 \phi_2^\dagger - \phi^2 \phi_1^\dagger - \hat{\phi}^{\dagger 1'} \hat{\phi}_{2'} - \hat{\phi}^{\dagger 2'} \hat{\phi}_{1'}), \\ R^{56} &= \int d^2x \text{Tr}(-\psi^{\dagger 2'} \psi_{2'} + \psi^{\dagger 1'} \psi_{1'} + \hat{\psi}^2 \hat{\psi}_2^\dagger - \hat{\psi}^1 \hat{\psi}_1^\dagger + \phi^1 \phi_1^\dagger - \phi^2 \phi_2^\dagger + \hat{\phi}^{\dagger 1'} \hat{\phi}_{1'} - \hat{\phi}^{\dagger 2'} \hat{\phi}_{2'}). \end{aligned} \quad (4.9)$$

We cannot construct a dynamical SUSY charge Q_2 and hence there is no superconformal generator S . This gives us an example of non-relativistic superconformal field theories with no superconformal charges.

⁸Since there are two relations: $M = Q_{R_1} - Q_{R_2}$ and $Q_B + Q_F = Q_{R_1} + Q_{R_2}$, the total symmetry is $U(2) \times U(2) \times U(1)_F$. In addition, a particular $U(1) \times U(1)$ is a part of the gauge symmetry.

4.2 4 SUSY limit

We take the ansatz for the non-relativistic limit of scalars as

$$X^a = \frac{1}{\sqrt{2m}} e^{-imt} \phi^a, \quad X^{a'} = \frac{1}{\sqrt{2m}} e^{imt} \hat{\phi}^{*a'}, \quad (4.10)$$

and fermions as

$$\Psi_a = e^{-imt} \psi_a, \quad \Psi_{a'} = e^{imt} \sigma_2 \hat{\psi}_{a'}^*. \quad (4.11)$$

The Dirac equation for $\Psi_{a'}$ gives slightly different results from those in section 3:

$$\Psi_{a'} = e^{imt} \begin{pmatrix} \frac{iD_+}{2m} \hat{\psi}_{a'}^* \\ i\hat{\psi}_{a'}^* \end{pmatrix}. \quad (4.12)$$

The action is given by $S_{\text{CS}} + S_{\text{kin}} + S_{\text{bos}} + S_{\text{fer}}$, where S_{CS} is the same as in (3.10) while the kinetic term is given by

$$S_{\text{kin}} = \int dt d^2x \left[i\text{Tr}(\phi_a^\dagger D_0 \phi^a + \hat{\phi}^{\dagger a'} D_0 \hat{\phi}_{a'}) - \frac{1}{2m} \text{Tr}(D_i \phi_a^\dagger D_i \phi^a + D_i \hat{\phi}^{\dagger a'} D_i \hat{\phi}_{a'}) \right. \\ \left. + i\text{Tr}(\psi^{\dagger a} D_0 \psi_a + \hat{\psi}_{a'}^\dagger D_0 \hat{\psi}^{a'}) + \frac{1}{2m} \text{Tr}(\psi^{\dagger a} D_- D_+ \psi_a + \hat{\psi}_{a'}^\dagger D_+ D_- \hat{\psi}^{a'}) \right]. \quad (4.13)$$

Now, ϕ^a and ψ_a transform as (N, \bar{N}) under $U(N) \times U(N)$ whereas $\hat{\phi}_{a'}$ and $\hat{\psi}^{a'}$ transform as (\bar{N}, N) . This is equivalent to the non-relativistic limit studied in [22].

The leading bosonic potential that will survive in the conformal limit is

$$S_{\text{bos}} = \frac{\pi}{km} \int dt d^2x \text{Tr}(\phi^a \phi_{[a}^\dagger \phi^b \phi_{b]}^\dagger - \hat{\phi}^{\dagger a'} \hat{\phi}_{[a'} \hat{\phi}^{\dagger b'} \hat{\phi}_{b']}). \quad (4.14)$$

The fermionic potential comes from the non-relativistic limit of (2.3):

$$S_{\text{fer}} = \frac{\pi}{km} \int dt d^2x \text{Tr} \left[(\phi_a^\dagger \phi^a + \hat{\phi}_{a'} \hat{\phi}^{\dagger a'}) (\psi^{\dagger b} \psi_b + \hat{\psi}^{b'} \hat{\psi}_{b'}^\dagger) \right. \\ \left. + (\phi^a \phi_a^\dagger + \hat{\phi}^{\dagger a'} \hat{\phi}_{a'}) (\psi_b \psi^{\dagger b} + \hat{\psi}_{b'} \hat{\psi}^{b'}) \right. \\ \left. - 2(\phi^a \phi_b^\dagger \psi_a \psi^{\dagger b} + \hat{\phi}^{\dagger a'} \hat{\phi}_{b'} \hat{\psi}_{a'}^\dagger \hat{\psi}^{b'}) - 2(\phi_a^\dagger \phi^b \psi^{\dagger a} \psi_b + \hat{\phi}_{a'} \hat{\phi}^{\dagger b'} \hat{\psi}^{a'} \hat{\psi}_{b'}^\dagger) \right]. \quad (4.15)$$

Let us study the bosonic symmetry of the theory. The theory possesses the full Schrödinger symmetry and $SU(2) \times SU(2)$ R-symmetry acting on indices a and a' . In addition, the theory is invariant under $U(1)_B$ and $U(1)_F$ generated by $Q_B(\phi^a, \hat{\phi}_{a'}, \psi_a, \hat{\psi}^{a'}) = (1, -1, 0, 0)$ and $Q_F(\phi^a, \hat{\phi}_{a'}, \psi_a, \hat{\psi}^{a'}) = (0, 0, 1, -1)$. Furthermore, because ϵ^{ab} and $\epsilon^{a'b'}$ do not appear in the action, the $SU(2) \times SU(2)$ symmetry is enhanced to $U(2) \times U(2)$ with additional $U(1)_R$ charge generated by $Q_{R_1}(\phi^a, \hat{\phi}_{a'}, \psi_a, \hat{\psi}^{a'}) = (1, 0, -1, 0)$ and $Q_{R_2}(\phi^a, \hat{\phi}_{a'}, \psi_a, \hat{\psi}^{a'}) = (0, 1, 0, -1)$.⁹

⁹There is one relation between $U(1)$ charges: $Q_{R_1} - Q_{R_2} = Q_B - Q_F$, so the total symmetry is $U(2) \times U(2) \times U(1)_F \times U(1)_M$. In other words, the $U(1)$ symmetries are generated by all the independent rotations of $(\phi^a, \hat{\phi}_{a'}, \psi_a, \hat{\psi}^{a'})$. A particular combination of $U(1) \times U(1)$ is a part of the gauge symmetry.

While the bosonic sector has a larger symmetry than the limit discussed in section 3, the number of supersymmetry is reduced. This is due to the fact that the supersymmetries generated by Γ^{3-6} do not act on the fields non-trivially any longer because the particle cannot transform into anti-particle in the non-relativistic limit. The only non-trivial SUSY transformations are generated by Γ^1 and Γ^2 . The kinematical SUSY charges are

$$\begin{aligned} Q_1(\equiv Q_1^1 + iQ_1^2) &= \sqrt{2mi} \int d^2x \text{Tr} \left(i\phi_1^\dagger \psi_2 - i\phi_2^\dagger \psi_1 \right), \\ \hat{Q}_1(\equiv Q_1^1 - iQ_1^2) &= \sqrt{2mi} \int d^2x \text{Tr} \left(-\hat{\phi}_1 \hat{\psi}_2^\dagger + \hat{\phi}_2 \hat{\psi}_1^\dagger \right), \end{aligned} \quad (4.16)$$

and there is no dynamical SUSY. As a consequence, there is no superconformal symmetry. The anti-commutation relations are

$$\{Q_1^*, Q_1\} = 2M_P, \quad \{\hat{Q}_1^*, \hat{Q}_1\} = 2\hat{M}_A, \quad \{Q_1, \hat{Q}_1\} = \{Q_1^*, \hat{Q}_1\} = 0, \quad (4.17)$$

where M_P is the mass operator for particles, and \hat{M}_A is the mass operator for anti-particles.

4.3 0 SUSY limit

We can construct a non-supersymmetric theory by taking the non-relativistic ansatz

$$X^A = \frac{1}{\sqrt{2m}} e^{-imt} \phi^A \quad (4.18)$$

and

$$\Psi_A = e^{imt} \sigma_2 \hat{\psi}_A^*. \quad (4.19)$$

It is clear that since the bosons are all particles and fermions are all anti-particles, there is no non-trivial supersymmetry acting on the non-relativistic theory.

Without writing down the action explicitly, we just point out that the bosonic symmetry is given by the Schrödinger algebra with global $SU(2) \times SU(2) \times U(1)_B \times U(1)_F$ symmetries. Due to the lack of the supersymmetry, however, it is quite probable that the model breaks the conformal invariance at the quantum level. Conformal invariance of the non-relativistic Chern-Simons-Matter theory has been discussed in [31–33].

5 Discussion and summary

In this paper, we have studied various non-relativistic limits of the $\mathcal{N} = 6$ superconformal field theories and constructed different non-relativistic conformal field theories. While the kinematical SUSY is easy to obtain, the emergence of the dynamical SUSY is non-trivial. We need a specific combination of the relativistic supersymmetry whose leading order supersymmetry transformation vanishes in the non-relativistic limit.

One may try to obtain more supersymmetries by starting with Bagger-Lambert $\mathcal{N} = 8$ supersymmetric Chern-Simons theory [2, 3]. Again it is not so difficult to construct the limit where only the kinematical SUSY remains while it is still an open question whether we could obtain more dynamical supersymmetries there.

Given a new non-relativistic superconformal algebra, one could define a (non-relativistic) superconformal index [22], and compute it from the explicit theory we have constructed in this paper. The superconformal algebras we have obtained in this paper have a non-trivial involutive anti-automorphism, so it is straight-forward to define a new class of indices.

Finally, the supergravity dual of the non-relativistic limit of the ABJM theory is of most importance for a future study. The existence of several different non-relativistic limits, as we have discussed in this paper, suggests that corresponding different non-relativistic limits should also exist in the dual supergravity solution. It would be very interesting to pursue this direction further. Some related supergravity backgrounds with Schrödinger (super)symmetry have been studied in [34–51].

Acknowledgments

We would like to thank S. Ryu for fruitful discussions. The work of NY was supported in part by the National Science Foundation under Grant No. PHY05-55662 and the UC Berkeley Center for Theoretical Physics. The work of MS was supported in part by the Grant-in-Aid for Scientific Research (19540324) from the Ministry of Education, Science and Culture, Japan. The work of KY was supported in part by the National Science Foundation under Grant No. PHY05-51164 and JSPS Postdoctoral Fellowships for Research Abroad.

A Spinor convention

We take the $(-, +, +)$ metric convention and chiral representation of the gamma matrix in $(1 + 2)$ dimension: $\gamma^\mu = (i\sigma_3, \sigma_1, -\sigma_2)$. They satisfy the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$. The Dirac conjugation is given by $\bar{\psi} = \psi^\dagger \gamma^0 = \psi^\dagger i\sigma^3$. The corresponding scalar product is $\bar{\psi}\psi \equiv i\psi_\alpha^* \sigma_3^{\alpha\beta} \psi_\beta$. We can define a raised spinor by $\psi^\alpha = \epsilon^{\alpha\beta} \psi_\beta = i\sigma_2^{\alpha\beta} \psi_\beta$ so that $\chi\psi \equiv \chi^\alpha \psi_\alpha$ is a Lorentz scalar. Similarly we define $\psi^\dagger \chi^\dagger \equiv \epsilon^{\alpha\beta} \psi_\beta^\dagger \chi_\alpha^\dagger = -(\chi\psi)^*$.

In this chiral basis, the Majorana condition is imposed by $\alpha\sigma_1\psi^* = \psi$ with $|\alpha|^2 = 1$. We choose $\alpha = -i$ with no loss of generality.

B Consistency of the truncation

In this appendix, we address the consistency of the non-relativistic truncation studied in the main text. When we substitute the non-relativistic ansatz with both particles and anti-particles into the relativistic action, we have non-trivial interactions that might induce inconsistency.

Having Non-Abelian ordering of the operators and index structure suppressed, which are irrelevant for this study, we have the following interactions in the relativistic theory

$$\begin{aligned}
 & (\phi_a \phi_{b'}^* + \hat{\phi}_c^* \hat{\phi}_{d'}) (\psi_e \psi_{f'}^* + \hat{\psi}_g^* \hat{\psi}_{h'}) + \text{c.c.}, \\
 & \phi_a^* \hat{\phi}_b \psi_c \hat{\psi}_d^* + \text{c.c.}, \quad \phi_{a'}^* \hat{\phi}_{b'} \psi_{c'} \hat{\psi}_{d'}^* + \text{c.c.}, \\
 & \phi_{a'}^* \psi_{b'} \hat{\phi}_c \hat{\psi}_d^* + \text{c.c.}, \quad \phi_{a'}^* \psi_b \hat{\phi}_{c'} \hat{\psi}_d^* + \text{c.c.}, \\
 & \phi_{a'}^* \psi_b \hat{\phi}_c \hat{\psi}_{d'}^* + \text{c.c.}, \quad \phi_a^* \psi_{b'} \hat{\phi}_{c'} \hat{\psi}_d^* + \text{c.c.}, \\
 & \phi_a^* \psi_{b'} \hat{\phi}_c \hat{\psi}_{d'}^* + \text{c.c.}, \quad \phi_a^* \psi_b \hat{\phi}_{c'} \hat{\psi}_{d'}^* + \text{c.c.} .
 \end{aligned} \tag{B.1}$$

As discussed in [23], we can impose either the strong condition, which means the conservation of the particle number, or the weak condition, which means the consistency at the level of classical equation of motion. The former is strong because there could be no quantum creation of particles, but the latter truncation is still consistent as a classical theory because it does not provide any source for discarded fields.

We see that the PPPP truncation (section 3) is consistent under the strong condition while PAAP (section 4.1), PAPA (section 4.2) and PPAA (section 4.3) truncations are only consistent under the weak condition. We could imagine the truncation which does not satisfy any condition such as PPPA truncation. While there is no problem in finding classical Schrödinger invariant field theories from such a construction, the supersymmetry is typically broken.

References

- [1] A. Gustavsson, *Algebraic structures on parallel M2-branes*, *Nucl. Phys. B* **811** (2009) 66 [[arXiv:0709.1260](#)] [[SPIRES](#)].
- [2] J. Bagger and N. Lambert, *Gauge Symmetry and Supersymmetry of Multiple M2-Branes*, *Phys. Rev. D* **77** (2008) 065008 [[arXiv:0711.0955](#)] [[SPIRES](#)].
- [3] J. Bagger and N. Lambert, *Comments On Multiple M2-branes*, *JHEP* **02** (2008) 105 [[arXiv:0712.3738](#)] [[SPIRES](#)].
- [4] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [[arXiv:0806.1218](#)] [[SPIRES](#)].
- [5] M. Fujita, W. Li, S. Ryu and T. Takayanagi, *Fractional Quantum Hall Effect via Holography: chern-Simons, Edge States and Hierarchy*, [arXiv:0901.0924](#) [[SPIRES](#)].
- [6] O. Aharony, O. Bergman and D.L. Jafferis, *Fractional M2-branes*, *JHEP* **11** (2008) 043 [[arXiv:0807.4924](#)] [[SPIRES](#)].
- [7] A. Giveon and D. Kutasov, *Seiberg Duality in Chern-Simons Theory*, *Nucl. Phys. B* **812** (2009) 1 [[arXiv:0808.0360](#)] [[SPIRES](#)].
- [8] M. Sakaguchi and K. Yoshida, *Super Schrödinger algebra in AdS/CFT*, *J. Math. Phys.* **49** (2008) 102302 [[arXiv:0805.2661](#)] [[SPIRES](#)].
- [9] M. Sakaguchi and K. Yoshida, *More super Schrödinger algebras from PSU(2, 2|4)*, *JHEP* **08** (2008) 049 [[arXiv:0806.3612](#)] [[SPIRES](#)].

- [10] C.R. Hagen, *Scale and conformal transformations in galilean-covariant field theory*, *Phys. Rev. D* **5** (1972) 377 [SPIRES].
- [11] U. Niederer, *The maximal kinematical invariance group of the free Schrödinger equation*, *Helv. Phys. Acta* **45** (1972) 802 [SPIRES].
- [12] M. Perroud, *Projective Representations of the Schrödinger Group*, *Helv. Phys. Acta* **50** (1977) 233 [SPIRES].
- [13] A.O. Barut and B.-W. Xu, *Conformal covariance and the probability interpretation of wave equations*, *Phys. Lett. A* **82** (1981) 218 [SPIRES].
- [14] V. Hussin and M. Jacques, *On nonrelativistic conformal symmetries and invariant tensor fields*, *J. Phys. A* **19** (1986) 3471 [SPIRES].
- [15] R. Jackiw and S.-Y. Pi, *Classical and quantal nonrelativistic Chern-Simons theory*, *Phys. Rev. D* **42** (1990) 3500 [Erratum *ibid.* **D 48** (1993) 3929] [SPIRES].
- [16] R. Jackiw and S.-Y. Pi, *Finite and infinite symmetries in (2+1)-dimensional field theory*, [hep-th/9206092](#) [SPIRES].
- [17] M. Henkel, *Schrödinger invariance in strongly anisotropic critical systems*, *J. Stat. Phys.* **75** (1994) 1023 [[hep-th/9310081](#)] [SPIRES].
- [18] V.K. Dobrev, H.D. Doebner and C. Mrugalla, *Lowest weight representations of the Schrödinger algebra and generalized heat equations*, *Rep. Math. Phys.* **39** (1997) 201.
- [19] T. Mehen, I.W. Stewart and M.B. Wise, *Conformal invariance for non-relativistic field theory*, *Phys. Lett. B* **474** (2000) 145 [[hep-th/9910025](#)] [SPIRES].
- [20] Y. Nishida and D.T. Son, *Nonrelativistic conformal field theories*, *Phys. Rev. D* **76** (2007) 086004 [[arXiv:0706.3746](#)] [SPIRES].
- [21] M. Leblanc, G. Lozano and H. Min, *Extended superconformal Galilean symmetry in Chern-Simons matter systems*, *Ann. Phys.* **219** (1992) 328 [[hep-th/9206039](#)] [SPIRES].
- [22] Y. Nakayama, *Index for Non-relativistic Superconformal Field Theories*, *JHEP* **10** (2008) 083 [[arXiv:0807.3344](#)] [SPIRES].
- [23] Y. Nakayama, S. Ryu, M. Sakaguchi and K. Yoshida, *A family of super Schrödinger invariant Chern-Simons matter systems*, *JHEP* **01** (2009) 006 [[arXiv:0811.2461](#)] [SPIRES].
- [24] Y. Nakayama and M. Sakaguchi and K. Yoshida, *Interacting SUSY-singlet matter in non-relativistic Chern-Simons theory*, [arXiv:0812.1564](#) [SPIRES].
- [25] K. Hosomichi, K.-M. Lee, S. Lee, S. Lee and J. Park, *$N=4$ Superconformal Chern-Simons Theories with Hyper and Twisted Hyper Multiplets*, *JHEP* **07** (2008) 091 [[arXiv:0805.3662](#)] [SPIRES].
- [26] K. Hosomichi, K.-M. Lee, S. Lee, S. Lee and J. Park, *$N=5,6$ Superconformal Chern-Simons Theories and $M2$ -branes on Orbifolds*, *JHEP* **09** (2008) 002 [[arXiv:0806.4977](#)] [SPIRES].
- [27] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, *A Massive Study of $M2$ -brane Proposals*, *JHEP* **09** (2008) 113 [[arXiv:0807.1074](#)] [SPIRES].
- [28] D. Gaiotto, S. Giombi and X. Yin, *Spin Chains in $N = 6$ Superconformal Chern-Simons-Matter Theory*, [arXiv:0806.4589](#) [SPIRES].
- [29] S. Terashima, *On $M5$ -branes in $N = 6$ Membrane Action*, *JHEP* **08** (2008) 080 [[arXiv:0807.0197](#)] [SPIRES].

- [30] M.A. Bandres, A.E. Lipstein and J.H. Schwarz, *Studies of the ABJM Theory in a Formulation with Manifest $SU(4)$ R-Symmetry*, *JHEP* **09** (2008) 027 [[arXiv:0807.0880](#)] [[SPIRES](#)].
- [31] O. Bergman and G. Lozano, *Aharonov-Bohm scattering, contact interactions and scale invariance*, *Ann. Phys.* **229** (1994) 416 [[hep-th/9302116](#)] [[SPIRES](#)].
- [32] S.-J. Kim and C.-k. Lee, *Quantum Description of Anyons: role of Contact Terms*, *Phys. Rev. D* **55** (1997) 2227 [[hep-th/9606054](#)] [[SPIRES](#)].
- [33] M.O. de Kok and J.W. van Holten, *The Role of Conformal Symmetry in the Jackiw-Pi Model*, *Nucl. Phys. B* **805** (2008) 545 [[arXiv:0806.3358](#)] [[SPIRES](#)].
- [34] D.T. Son, *Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry*, *Phys. Rev. D* **78** (2008) 046003 [[arXiv:0804.3972](#)] [[SPIRES](#)].
- [35] K. Balasubramanian and J. McGreevy, *Gravity duals for non-relativistic CFTs*, *Phys. Rev. Lett.* **101** (2008) 061601 [[arXiv:0804.4053](#)] [[SPIRES](#)].
- [36] W.D. Goldberger, *AdS/CFT duality for non-relativistic field theory*, *JHEP* **03** (2009) 069 [[arXiv:0806.2867](#)] [[SPIRES](#)].
- [37] J.L.F. Barbon and C.A. Fuertes, *On the spectrum of nonrelativistic AdS/CFT*, *JHEP* **09** (2008) 030 [[arXiv:0806.3244](#)] [[SPIRES](#)].
- [38] W.-Y. Wen, *AdS/NRCFT for the (super) Calogero model*, [arXiv:0807.0633](#) [[SPIRES](#)].
- [39] C.P. Herzog, M. Rangamani and S.F. Ross, *Heating up Galilean holography*, *JHEP* **11** (2008) 080 [[arXiv:0807.1099](#)] [[SPIRES](#)].
- [40] J. Maldacena, D. Martelli and Y. Tachikawa, *Comments on string theory backgrounds with non-relativistic conformal symmetry*, *JHEP* **10** (2008) 072 [[arXiv:0807.1100](#)] [[SPIRES](#)].
- [41] A. Adams, K. Balasubramanian and J. McGreevy, *Hot Spacetimes for Cold Atoms*, *JHEP* **11** (2008) 059 [[arXiv:0807.1111](#)] [[SPIRES](#)].
- [42] S. Kachru, X. Liu and M. Mulligan, *Gravity Duals of Lifshitz-like Fixed Points*, *Phys. Rev. D* **78** (2008) 106005 [[arXiv:0808.1725](#)] [[SPIRES](#)].
- [43] S.A. Hartnoll and K. Yoshida, *Families of IIB duals for nonrelativistic CFTs*, *JHEP* **12** (2008) 071 [[arXiv:0810.0298](#)] [[SPIRES](#)].
- [44] M. Schvellinger, *Kerr-AdS black holes and non-relativistic conformal QM theories in diverse dimensions*, *JHEP* **12** (2008) 004 [[arXiv:0810.3011](#)] [[SPIRES](#)].
- [45] L. Mazzucato, Y. Oz and S. Theisen, *Non-relativistic Branes*, [arXiv:0810.3673](#) [[SPIRES](#)].
- [46] M. Rangamani, S.F. Ross, D.T. Son and E.G. Thompson, *Conformal non-relativistic hydrodynamics from gravity*, *JHEP* **01** (2009) 075 [[arXiv:0811.2049](#)] [[SPIRES](#)].
- [47] A. Adams, A. Maloney, A. Sinha and S.E. Vazquez, *1/N Effects in Non-Relativistic Gauge-Gravity Duality*, *JHEP* **03** (2009) 097 [[arXiv:0812.0166](#)] [[SPIRES](#)].
- [48] M. Taylor, *Non-relativistic holography*, [arXiv:0812.0530](#) [[SPIRES](#)].
- [49] A. Donos and J.P. Gauntlett, *Supersymmetric solutions for non-relativistic holography*, *JHEP* **03** (2009) 138 [[arXiv:0901.0818](#)] [[SPIRES](#)].
- [50] A. Ghodsi and M. Alishahiha, *Non-relativistic D3-brane in the presence of higher derivative corrections*, [arXiv:0901.3431](#) [[SPIRES](#)].
- [51] A. Akhavan, M. Alishahiha, A. Davody and A. Vahedi, *Fermions in non-relativistic AdS/CFT correspondence*, [arXiv:0902.0276](#) [[SPIRES](#)].